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PLATE IN THE FORM OF AN INFINITE STRIP ON AN ELASTIC HALF-SPACE

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A method is given for solving the problem about the contact of a thin plate in the form of an infinite strip lying on an elastic frictionless half-space in a three-dimensional formulation. This method reduces the problem to the solution of an infinite system of algebraic equations with a completely continuous form.

The corresponding plane problem has been examined by a number of authors [1 — 6]. The results obtained in [3 — 6] should be considerd most complete. This same problem for a linearly deformable base of general type was examined in [7 — 9].

1. The problem of bending of a plate lying on an elastic frictionless half-space reduces to the following system of equations:

$$D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 w(x,y)=g(x,y)-r(x,y) \tag{1.1}$$

$$\frac{1-\nu^2}{\pi E}\iint_S \frac{r(\xi,\eta)\,d\xi d\eta}{\sqrt{(x-\xi)^2+(y-\eta)^2}}=w(x,y)$$

Here D is the cylindrical stiffness of the plate, $w(x, y)$ is the deflection, $g(x, y)$ is the vertical load, $r(\xi, \eta)$ is the reaction of the foundation, and S is the contact domain.

To solve this system, it is important to be able to solve the second equation in (1.1), the fundamental integral equation of the contact problem of a stamp in the shape of the contact domain S under the condition that the surface of the stamp base is curved according to the law $z = w(x, y)$. In general, the function $w(x, y)$ is unknown, hence a general (or at least sufficiently general) solution of the mentioned contact problem must be able to be found. The general solution of the contact problem for a strip stamp, obtained in [10], will be used to solve the problem on the bending of a plate in the shape of a strip of width 2.

Let us assume that a load represented in the form

$$g(x, y) = F^{-1}[g^*(x, \lambda)], \quad g^*(x, \lambda) = F[g(x, y)]$$

acts on the plate, where $F[u]$ and $F^{-1}[u]$ are the direct and inverse Fourier transforms. We apply the Fourier transform to the functions $w(x, y)$ and $r(x, y)$. Then the system (1.1) will be satisfied under the condition that $\{w^*(x, \lambda), r^*(x, \lambda)\}$ is the solution of the following system of equations:

$$D\left(\frac{d^2}{dx^2}-\lambda^2\right)^2 w^*(x,\lambda)-g^*(x,\lambda)+r^*(x,\lambda)=0 \tag{1.2}$$

$$\frac{1-\nu^2}{\pi E}\int_{-1}^{1} r^*(\xi,\lambda)K_0(\lambda|x-\xi|)\,d\xi=w^*(x,\lambda) \quad (|x|\leqslant 1)$$

The solution of the last equation is [10]

$$r^*(x,\lambda) = \frac{-E}{(1-\nu^2)\,|\sin\eta|}\sum_{m=0}^{\infty}\gamma_m \frac{\mathrm{Fek}'_m(0,-q)}{\mathrm{Fek}_m(0,-q)}\,\mathrm{ce}_m(\eta,-q) \qquad (1.3)$$

$$0 \leqslant \eta \leqslant \pi$$

Here $\mathrm{Fek}_m(x, -q)$, $\mathrm{ce}_m(x, -q)$ are the known Mathieu functions [11], γ_m are coefficients of the expansion of the function $w^*(x, \lambda)$ in Fourier series in periodic Mathieu functions

$$w^*(x,\lambda) = \sum_{m=0}^{\infty}\gamma_m\,\mathrm{ce}_m(\eta,-q) \quad \left(\eta = \arccos x, \quad q = \frac{1}{4}\lambda^2\right) \qquad (1.4)$$

We consider a plate whose edges are free. Then the deflection $w^*(x, \lambda)$ should satisfy the conditions

$$\frac{d^2w^*}{dx^2} - \nu_0\lambda^2 w^* = 0, \quad \frac{d^3w^*}{dx^3} - \lambda^2(1-\nu_0)\frac{dw^*}{dx} = 0 \qquad \text{for } x = \pm 1 \qquad (1.5)$$

(ν_0 is the plate Poisson's ratio).

The solution of the first equation in (1.2) is written as

$$w^*(x, \lambda) = w_1(x, \lambda) + w_0(x, \lambda) \qquad (1.6)$$

where $w_1(x, \lambda)$ is a particular solution of the inhomogeneous equation and $w_0(x, \lambda)$ is the general solution of the homogeneous equation.

We define the function $w_0(x, \lambda)$ so that the conditions

$$\frac{d^2w_0}{dx^2} - \nu_0\lambda^2 w_0 = \nu_0\lambda^2 w_1 - \frac{d^2w_1}{dx^2}$$

$$\frac{d^3w_0}{dx^3} - (2-\nu_0)\lambda^2\frac{dw_0}{dx} = (2-\nu_0)\lambda^2\frac{dw_1}{dx} - \frac{d^3w_1}{dx^3} \qquad \text{for } x = \pm 1$$

would be satisfied. This function is found by elementary means after which the solution $w_1(x, \lambda)$ is found.

Let us define the solution $w_1(x, \lambda)$ by the conditions

$$w_1(x, \lambda) = d^2w_1 / dx^2 = 0 \qquad \text{for } x = \pm 1 \qquad (1.7)$$

Then the function $w^*(x, \lambda)$ defined by (1.6) will satisfy the conditions (1.5).

Henceforth, we shall consider that the solution w_0 is known. The problem is to find the function $w_1(x, \lambda)$ satisfying the system (1.2) and the boundary conditions (1.7).

2. Let $G(x, \xi)$ be the Green's function of the boundary value problem

$$\frac{d^4y}{dx^4} - 2\lambda^2\frac{d^2y}{dx^2} = 0, \quad y(\pm 1) = y''(\pm 1) = 0$$

Such function can easily be constructed and has the form

$$G(x,\xi) = \frac{1}{4\lambda^2}\left\{1 - x\xi - |x-\xi| + \frac{1}{\sqrt{2}\,\lambda}\,\mathrm{sh}\sqrt{2}\,\lambda\,|x-\xi| + \frac{\mathrm{ch}\sqrt{2}\,\lambda(x+\xi) - \mathrm{ch}\,2\sqrt{2}\,\lambda\,\mathrm{ch}\sqrt{2}\,\lambda(x-\xi)}{\sqrt{2}\,\lambda\,\mathrm{sh}\,2\sqrt{2}\,\lambda}\right\} \qquad (2.1)$$

By using the Green's function (2.1), we represent the first equation in (1.2) as an equivalent integral relation

$$w_1(x,\lambda) = k_0 \int_{-1}^{1} G(x,\xi)\,[g^*(\xi,\lambda) - r^*(\xi,\lambda)]\,d\xi - \lambda^4 \int_{-1}^{1} G(x,\xi)\,w_1(\xi,\lambda)\,d\xi \quad (k_0 = D^{-1}) \tag{2.2}$$

Representing the function $w_1(x, \lambda)$ as the expansion (1.4) and using the solution (1.3) corresponding to this expansion, we obtain from (2.2)

$$\sum_{m=0}^{\infty} \gamma_m \,\mathrm{ce}_m(\eta, -q) = k_1 \sum_{m=0}^{\infty} \gamma_m \frac{\mathrm{Fek}_m'(0,-q)}{\mathrm{Fek}_m(0,-q)} \times \int_{-1}^{1} G(x,\xi) \frac{\mathrm{ce}_m(t,-q)}{|\sin t|}\,d\xi - \lambda^4 \sum_{m=0}^{\infty} \gamma_m \int_{-1}^{1} G(x,\xi)\,\mathrm{ce}_m(t,-q)\,d\xi + f(\eta) \tag{2.3}$$

$$f(\eta) = k_0 \int_{-1}^{1} G(x,\xi)\, g^*(\xi,\lambda)\,d\xi$$

$$t = \arccos \xi, \quad \eta = \arccos x, \quad k_1 = Ek_0 / (1 - \nu^2)$$

After multiplying (2.3) by $\mathrm{ce}_k(\eta, -q)$ and subsequent integration over the interval $(0, \pi)$, we find

$$\pi_k \gamma_k = \sum_{m=0}^{\infty} T_k^{(m)} \gamma_m + \sum_{m=0}^{\infty} K_k^{(m)} \gamma_m + \alpha_k \tag{2.4}$$

$$(\pi_0 = \pi, \ \pi_k = \pi / 2, \ k \geqslant 1)$$

The matrix coefficients and free terms in the infinite system obtained are found by means of the formulas

$$T_k^{(m)} = k_1 \frac{\mathrm{Fek}_m'(0,-q)}{\mathrm{Fek}_m(0,-q)} \int_0^{\pi} \mathrm{ce}_k(\eta, -q)\,d\eta \int_{-1}^{1} G(x,\xi) \frac{\mathrm{ce}_m(t,-q)}{|\sin t|}\,d\xi \tag{2.5}$$

$$K_k^{(m)} = -\lambda^4 \int_0^{\pi} \mathrm{ce}_k(\eta,-q)\,d\eta \int_{-1}^{1} G(x,\xi)\,\mathrm{ce}_m(t,-q)\,d\xi$$

$$\alpha_k = k_0 \int_0^{\pi} \mathrm{ce}_k(\eta,-q)\,d\eta \int_{-1}^{1} G(x,\xi)\, g^*(\xi,\lambda)\,d\xi$$

$$(\xi = \cos t, \ x = \cos \eta)$$

We represent the Green's function (2.1) as a uniformly convergent bilinear expansion in the products of eigenfunctions of the self-adjoint boundary value problem

$$\frac{d^4 y}{dx^4} - 2\lambda^2 \frac{d^2 y}{dx^2} - \mu y = 0, \quad y(\pm 1) = y''(\pm 1) = 0$$

The required expansion is

$$G(x,\xi) = \sum_{k=1}^{\infty} \frac{\sin p_{2k} x \sin p_{2k} \xi}{\mu_{2k}} + \sum_{k=0}^{\infty} \frac{\cos p_{2k+1} x \cos p_{2k+1} \xi}{\mu_{2k+1}} \tag{2.6}$$

where the eigenvalues μ_k are determined by the formula

$$\mu_k = p_k^4 + 2\lambda^2 p_k^2, \quad p_k = \frac{\pi}{2} k \quad (k \geqslant 1)$$

The expansion (2. 6) will be used to simplify the matrix coefficients and free members in the system (2. 4).

3. The representation (2. 6) permits obtaining an expansion of the function $G(x, \xi)$ in a series of Chebyshev polynomials

$$G(x, \xi) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij} T_i(x) T_j(\xi), \quad T_n(x) = \cos(n \arccos x) \tag{3.1}$$

We find the coefficients a_{ij} using the Fourier formulas from the theory of orthogonal functions. After some transformations we obtain

$$a_{00} = \frac{1}{4} \sum_{k=0}^{\infty} \frac{1}{\mu_{2k+1}} J_0(p_{2k+1}) J_0(p_{2k+1})$$

$$a_{2n\,0} = a_{0\,2n} = \frac{(-1)^n}{2} \sum_{k=0}^{\infty} \frac{1}{\mu_{2k+1}} J_0(p_{2k+1}) J_n(p_{2k+1})$$

$$a_{2m\,2n} = \frac{(-1)^{m+n}}{4} \sum_{k=0}^{\infty} \frac{1}{\mu_{2k+1}} J_{2m}(p_{2k+1}) J_{2n}(p_{2k+1})$$

$$a_{2m+1\,2n+1} = \frac{(-1)^{m+n}}{4} \sum_{k=1}^{\infty} \frac{1}{\mu_{2k}} J_{2m+1}(p_{2k}) J_{2n+1}(p_{2k})$$

These series converge uniformly in the subscripts m and n since $|J_n(x)| \leqslant 1$ for all x and n, and $\mu_k \sim (^1/_2\pi k)^4$ as $k \to \infty$.

The following assertion is valid: the coefficients a_{mn} satisfy the inequalities

$$|a_{mn}| \leqslant \frac{C}{m^2 n}, \quad |a_{mn}| \leqslant \frac{C}{mn^2}, \quad |a_{mn}| \leqslant \frac{C(m+n)}{m^2 n^2} \tag{3.2}$$

$(C = \mathrm{const}, \quad m, n \geqslant 2)$

For the proof we use the recurrence relation from Bessel function theory (z is real)

$$J_n(z) = \frac{z}{2n} [J_{n+1}(z) + J_{n-1}(z)] \tag{3.3}$$

It follows from (3. 3) that

$$|J_n(z)| \leqslant \begin{cases} \dfrac{|z|}{n}, & n \geqslant 1 \\ \dfrac{4}{3} \dfrac{z^2}{n^2}, & n \geqslant 2 \end{cases} \tag{3.4}$$

Let us consider the series $(\nu > 0)$

$$b_{mn} = \sum_{k=1}^{\infty} \frac{J_m(p_k)}{\mu_k^{5/8+\nu}} \frac{J_n(p_k)}{\mu_k^{3/8-\nu}}$$

Applying the Cauchy inequality, we find

$$b_{mn}^2 \leqslant \Sigma_1 \Sigma_2 \tag{3.5}$$

$$\Sigma_1 = \sum_{k=1}^{\infty} J_m^2(p_k) \mu_k^{-5/4-2\nu}, \quad \Sigma_2 = \sum_{k=1}^{\infty} J_n^2(p_k) \mu_k^{-3/4+2\nu}$$

For the series in the right side in (3.5) we obtain the following estimates by using the inequalities (3.4):

$$\Sigma_1 \leqslant \frac{16}{9m^4} \sum_{k=1}^{\infty} p_k^4 \mu_k^{-9/4-2\nu} = \frac{C_1^2(\lambda, \nu)}{m^4}$$

$$\Sigma_2 \leqslant \frac{1}{4n^2} \sum_{k=1}^{\infty} [J_{n+1}(p_k) + J_{n-1}(p_k)]^2 p_k^2 \mu_k^{-3/4+2\nu} \leqslant \frac{C_2^2(\lambda, \nu)}{n^2}$$

where the quantities $C_1^2(\lambda, \nu)$ and $C_2^2(\lambda, \nu)$ decrease with the increase in the parameter λ. The proof of the convergence of the last series follows from the inequality which is valid for large k

$$J_n^2\left(k\frac{\pi}{2}\right) = \frac{1}{\pi}\int_0^{\pi} J_0(k\pi \sin t)\cos 2nt\,dt \leqslant \frac{2}{\pi}\int_0^{\pi/2} |J_0(k\pi \sin t)|\,dt \leqslant$$

$$\frac{2}{\pi}\left(\frac{1}{\sqrt{k}} + K_0 \int_{1/\sqrt{k}}^{\pi/2} \frac{dt}{\sqrt{k\pi \sin t}}\right) \leqslant \frac{K_1}{\sqrt{k}} \qquad (K_0, K_1 = \text{const})$$

Thus, the inequality

$$|b_{mn}| \leqslant \frac{C_1 C_2}{m^2 n} \qquad (n \geqslant 1)$$

follows from (3.5), and the validity of the assertion (3.2) results.

The inequalities (3.2) permit establishment of the uniform and absolute convergence of the series (3.1). In fact, if $m > n$, then the second inequality in (3.2) results from the first one. Therefore, it can be considered that

$$|a_{mn}| \leqslant C / m^2 n \quad \text{for} \quad m > n$$

$$|a_{mn}| \leqslant C / n^2 m \quad \text{for} \quad n > m$$

Furthermore, taking account of the inequalities (3.2), we have

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} |a_{ij} T_i(x) T_j(\xi)| \leqslant \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} |a_{ij}| \leqslant 2\sum_{i=1}^{\infty}\sum_{j=1}^{i} \frac{C}{i^2 j} =$$

$$2C\sum_{i=1}^{\infty}\left[C + \ln i + O\left(\frac{1}{i}\right)\right] i^{-2} < \infty$$

(C is the Euler constant).

After these preliminary results, it is possible to proceed to the simplification of the matrix coefficients and free terms in the system of equations (2.4).

Substitution of the expansion (3.1) in the first two formulas in (2.5) results in the formulas

$$T_k^{(m)} = k_1 \frac{\mathrm{Fek}_m'(0, -q)}{\mathrm{Fek}_m(0, -q)} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \pi_i \pi_j C_i^{(k)} C_j^{(m)} a_{ij}$$

$$K_k^{(m)} = -\lambda^4 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \pi_i C_i^{(k)} D_j^{(m)} a_{ij}$$

$$D_j^{(m)} = {\sum_{r=0}^{\infty}}' C_r^{(m)} (-1)^{j+r}\left[\frac{1}{1-(j+r)^2} + \frac{1}{1-(j-r)^2}\right]$$

$$C_{2r}^{(2m)} = (-1)^{r+m} A_{2r}^{(2m)}, \qquad C_{2r+1}^{(2m+1)} = (-1)^{m+r} B_{2r+1}^{(2m+1)}$$

The prime at the summation means that members whose denominators vanish are omitted. $A_r^{(m)}$ and $B_r^{(m)}$ are coefficients of the expansion of periodic Mathieu functions in Fourier series in a trigonometric function system [11].

The free members of the infinite system (2.4) are computed by the formulas

$$\alpha_{2k} = k_0\pi(-1)^k \sum_{l=0}^{\infty} \frac{a_l}{\mu_{2l+1}} \sum_{r=0}^{\infty} A_{2r}^{(2k)} J_{2r}\left(l\pi + \frac{\pi}{2}\right) \tag{3.6}$$

$$\alpha_{2k+1} = k_0\pi(-1)^k \sum_{l=1}^{\infty} \frac{b_l}{\mu_{2l}} \sum_{r=0}^{\infty} B_{2r+1}^{(2k+1)} J_{2r+1}(l\pi)$$

$$a_l = \int_{-1}^{1} g^*(\xi, \lambda) \cos\frac{\pi}{2}(2l+1)\xi d\xi \tag{3.7}$$

$$b_l = \int_{-1}^{1} g^*(\ ,\lambda) \sin(l\pi\xi)\, d\xi$$

4. Let us turn to an investigation of the infinite system (2.4). We write it in the form of the operator equation

$$x = Ax + a \tag{4.1}$$

$$x = (\gamma_0, \gamma_1, \ldots, \gamma_m, \ldots)^T, \quad a = \pi_k^{-1}(\alpha_0^*, \alpha_1, \ldots, \alpha_m, \ldots)^T$$

$$A = \pi_k^{-1}[(T_k^{(m)}) + (K_k^{(m)})]$$

where $(a_k^{(m)})$ is the symbol of an infinite order matix corresponding to the system of equations (2.4), and T is the transpose symbol.

The following theorem holds: the system of equations (4.1) has the unique solution $x \in l^2$.

First, let us show that the operator A is completely continuous in the Hilbert space l^2. To do this it is sufficient to prove the convergence of the series

$$\sum_{m,\,k=0}^{\infty} |T_k^{(m)}|^2, \qquad \sum_{m,\,k=0}^{\infty} |K_k^{(m)}|^2 \tag{4.2}$$

Using asymptotic formulas for the coefficients $A_r^{(n)}$, $B_r^{(n)}$

$$\left.\begin{matrix} A_{n-2r}^{(n)} \\ B_{n-2r}^{(n)} \end{matrix}\right\} \sim \frac{(n-r-1)!}{r!\,(n-1)!}\left(\frac{q}{4}\right)^r, \quad 0 \leqslant 2r \leqslant n$$

$$\left.\begin{matrix} A_{n+2r}^{(n)} \\ B_{n+2r}^{(n)} \end{matrix}\right\} \sim \frac{(-1)^r\, n!}{r!\,(n+r)!}\left(\frac{q}{4}\right)^r, \quad r\, 0 \wedge\!\!\backslash$$

as well as the inequalities (3.2), we find

$$\left|\sum_{i+j=0}^{\infty} \pi_i\pi_j C_i^{(k)} C_j^{(m)} a_{ij}\right| \leqslant \frac{\text{const}_1}{m^2 k}, \quad (m, k) \to \infty \tag{4.3}$$

It can be obtained from the theory of expansions of the modified Mathieu functions $\text{Fek}_m(x, -q)$ that as $m \to \infty$ the following asymptotic formula is valid

$$\left| \frac{\mathrm{Fek}'_m(0, -q)}{\mathrm{Fek}_m(0, -q)} \right| \leqslant \delta m$$

where δ is some constant independent of m. The inequality (4.3) together with the last inequality permit us to establish that

$$|T_k^{(m)}| \leqslant \frac{\text{const}_2}{mk}, \quad (m, k) \to \infty \tag{4.4}$$

Convergence of the first series in (4.2) follows immediately from (4.4).

For the members of the second series we have

$$|K_k^{(m)}| \leqslant \lambda^4 \frac{\text{const}_3}{k^2} \left| \sum_{j=0}^{\infty}{}' \left[\frac{1}{1-(j+m)^2} + \frac{1}{1-(j-m)^2} \right] (1+j)^{-1} \right| \leqslant \lambda^4 \frac{\text{const}_4}{m^2 k^2}$$

Therefore, the second series in (4.2) also converges. It follows from the fact that the series (4.2) converge, that the operator A generated by the system (2.4) is completely continuous in the number space l^2. Moreover, we find from (3.6), as $k \to \infty$, that

$$|\alpha_k| \leqslant \text{const}_5/k^2$$

For the system (4.1) with operator A completely continuous in l^2 and free members belonging to l^2, the Hilbert alternative [12] is valid, from which the unique solvability of the infinite system is easily proved by relying on the unique solvability of the initial boundary value problem. Such an infinite system of linear algebraic equations can be solved by the method of reduction (truncation). Hence, the solutions of truncated systems tend to the exact solution of an infinite system as their order increases.

In conclusion, let us note that the method proposed for the solution of a problem of a strip plate can be extended to the I. Ia. Shtaerman problem for a combined foundation as well as to an elastic half-space in the case of systems of strip plates.

REFERENCES

1. Gorbunov-Posadov, M. I., Analysis of Structures on an Elastic Foundation. Gosstroiizdat, Moscow, 1953.
2. Klubin, P. I., Analysis of beam and circular slabs on an elastic foundation. Inzh. Sb., Vol. 12, 1952.
3. Ishkova, A. G., On the bending of a circular plate and infinite strip on an elastic half-space. Izv. Akad. Nauk SSSR, OTN, № 10, 1958.
4. Ishkova, A. G. and Tulaikov, A. N., Some problems on bending of plates on an elastic half-space. Inzh. Sb., Vol. 23, 1956.
5. Aleksandrov, V. M., Vorovich, I. I. and Solodovnik, M. D., Effective solution of problems on cylindrical bending of plates of finite width on an elastic half-space. Izv. Akad. Nauk SSSR, Mekhan. Tverd. Tela, № 4, 1973.
6. Aleksandrov, V. M. and Solodovnik, M. D., Asymptotic solution of the problem of cylindrical bending of a plate of finite width on an elastic half-space. Prikl. Mekhan., Vol. 10, № 7, 1974.
7. Aleksandrov, V. M. and Shatskikh, L. S., Universal program to compute the bending of beam slabs on a linearly-deformable foundation. Tr. Seventh All-Union Conference on the Theory of Shells and Plates. "Nauka", Moscow, 1970.
8. Shatskikh, L. S., On the analysis of bending of a slab on an elastic layer. Izv.

Akad.Nauk SSSR, Mekhan. Tverd. Tela, № 2, 1972.
9. Naumov, Iu. A. and Shevliakov, Iu. A., Bending of a beam slab on a multilayer foundation. Gidro-aeromekhanika, № 3, 1966.
10. Rvachev, V. L., The pressure of a punch, having the planform of a strip, upon an elastic half-space. PMM Vol. 20, № 2, 1956.
11. MacLachlan, N. V., Theory and Applications of Mathieu Functions. Izd. Inostr. Lit., Moscow, 1953.
12. Kantorovich, L. V. and Krylov, V. I., Approximate Methods of Higher Analysis. Interscience Publishers, New York and Groningen, 1964.

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ON THE DEFORMATION OF AN ELASTIC WEDGE PLATE REINFORCED BY A VARIABLE STIFFNESS BAR AND A METHOD OF SOLVING MIXED PROBLEMS

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The plane state of stress of an infinite elastic wedge reinforced by an infinite elastic bar along the bisectrix, whose stiffness varies as r^ω (r is the distance from the wedge apex), is considered. The problem is reduced to a first order difference equation for the displacement ω and is solved in closed form. The solution retains its meaning for $\omega = \pm\infty$, when the mentioned fundamental problem for the reinforced domain goes over into a mixed problem for the homogeneous domain. Therefore, the method proposed, which is applicable also to problems for rectangular, cylindrical and conical domains reinforced by bars, plates, circular slabs and shells of variable stiffness, is more general in specific respects than the Wiener-Hopf method.

Homogeneous [1—4] and inhomogeneous [5, 6] problems for an elastic wedge reinforced by constant stiffness bars have been studied earlier by using difference equations. Corresponding heat conduction and electromagnetic wave diffraction problems on a wedge have been solved in [7, 8], etc.

1. Let an elastic wedge-shaped plate $0 \leqslant r < \infty$, $-\alpha \leqslant \theta \leqslant \alpha$ of thickness h be welded completely along the bisectrix to an infinite elastic bar. The bar tensile and bending stiffnesses $2D_j(r)$ in the r, θ plane are expressed, respectively, by the equations

$$D_1(r) = \beta r + \gamma r^{1+\omega} \tag{1.1}$$

$$D_2(r) = \beta r^3 + \gamma r^{3+\omega} \tag{1.2}$$

where $\beta \geqslant 0$, $\gamma \geqslant 0$ and ω are any real numbers, where different numbers in (1.1) and (1.2) can be denoted by identical letters. The magnitudes of forces applied to the wedge or bar at the points $r = l_s$ will be denoted by the letters M, N, S with subscripts s, while the subscripts 0 and ∞ correspond to points of the wedge $r = 0$ and of the bar $r = \infty$ (see Fig. 1; the notation for the forces applied at the points $r = l_s$ and $r = \infty$